Theory overview of light meson decays

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Outline:
- Motivation
- Introductions: ChPT, R\(\chi\)PT
- Overview of lightest meson decays
- Summary
Motivation

Flavour physics and hadron processes: need to calculate QCD

We know how to use perturbation theory with weakly coupled forces

For strong interactions problem: the theory is written in terms of quarks and gluons, but experiments involve hadrons

For QCD, basically two different strategies:

- hadronic input either avoided (using other experiments, lattice)
- or calculated using the approximate symmetries of QCD
Effective field theory: basics

in general
- input: relevant degrees of freedom
- powercounting
- expansion parameter(s) $\Rightarrow$ range of validity

for low energy QCD: ChPT, see following...
ChPT – introduction: chiral symmetry

Starting global symmetry: $SU(3)_L \times SU(3)_R \times U(1)$

For Hamiltonian in chiral limit (masses of light quarks $=0$) one has

$$[Q_V^A, H_0] = 0$$

two possible realizations of symmetry:

[Vafa, Witten '84]:

In QCD ($\theta_{\text{QCD}} = 0$) vector subgroup cannot be
spontaneously broken

$$Q^a \mathcal{V}_0 = 0$$

"Wigner-Weyl realization" /Wigner: NP '63/

→ multiplet structure ($p, n, \Sigma, \Lambda, \Xi$)

[Coleman, Grossman '82]:

Having 'confinement' and 3 massless quarks

the axial symmetry is spontaneously broken

$$Q^a \mathcal{A}_0 \neq 0$$

"Nambu-Goldstone" /Nambu: NP '08/

→ 8 pseudoscalar massless Goldstone bosons

($\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, \eta$)

they form octet:

$$Q^a \mathcal{V}_\pi^b (p) = \text{if } abc \mathcal{V}_\pi^c (p)$$

The only particles we can predict nowadays from first principles of QCD by-hand!
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  \[ \rightarrow 8 \text{ pseudoscalar massless Goldston bosons} \]

  \( (\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, \eta) \)

  they form octet: \( Q^a_V |\pi^b(p)\rangle = i f^{abc} |\pi^c(p)\rangle \)
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For Hamiltonian in chiral limit (masses of light quarks $= 0$) one has

$$[Q_V, H_0] = 0$$

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Calculation within 2-flavour ChPT, even sector

\[ U = \sigma + i \frac{\Phi}{F} , \sigma^2 + \frac{\phi^2}{F^2} = 1 \, , \phi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix} = \phi^i \tau^i , \]

\[ u_\mu = i u^\dagger \partial_\mu U u^\dagger , \chi_+ = u^\dagger \chi u^\dagger + u \chi^\dagger u , \chi = 2B(\hat{m}) \, , \hat{m} = \frac{1}{2}(m_u + m_d) , \]

- **\( O(p^2) \)**

\[ \mathcal{L}_2 = \frac{F^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle . \]

- **\( O(p^4) \)**

\[ \mathcal{L}_4 = \frac{l_1}{4} \langle u_\mu u^\mu \rangle^2 + \frac{l_2}{4} \langle u_\mu u_\nu \rangle \langle u^\mu u^\nu \rangle + \frac{l_3}{16} \langle \chi_+ \rangle^2 + i \frac{l_4}{4} \langle u_\mu \chi_- \rangle - \frac{l_5}{2} \langle f^- \mu \nu f^- \mu \nu \rangle \\
+ i \frac{l_6}{2} \langle f^+ \mu \nu u^\mu u^\nu \rangle - \frac{l_7}{16} \langle \chi_- \rangle^2 \]

\[ l_i = l_i^r + \gamma_i (c \mu)^{d-4} \Lambda , \]

- **\( O(p^6) \)**

\[ \mathcal{L}_6 = \ldots + c_6 \langle \chi_+ h_{\mu \nu} h^{\mu \nu} \rangle + c_7 \langle u_\mu u^\mu \chi_+ \chi_+ \rangle + c_8 \langle u_\mu u^\mu \chi_+ \rangle \langle \chi_+ \rangle + c_9 \langle \chi_+ u_\mu \chi_+ u^\mu \rangle + \ldots \]

\[ c_i = \frac{(c \mu)^2(d-4)}{F^2} \left( c_i^r - \gamma_i^{(2)} \Lambda^2 - (\gamma_i^{(1)} + \gamma_i^{(L)}) \Lambda \right) . \]
Calculation within 2-flavour ChPT

NLO odd-intrinsic Lagrangian is given by [Bijnens, Girlanda, Talavera '02]
cf. also [Ebertshauser, Fearing and Scherer '01]

\[ \mathcal{L}_W^6 = \sum_{i=1}^{13} c_i^W o_i^W, \quad c_i^W = c_i^W r + \eta_i (c\mu)^{d-4} \Lambda, \quad (1) \]

<table>
<thead>
<tr>
<th>monomial ((o_i^W))</th>
<th>2-flavour</th>
<th>(384\pi^2F^2\eta_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\epsilon^{\mu\nu\alpha\beta}\langle x + [f_{-\mu\nu}, u_\alpha u_\beta]\rangle)</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>(\epsilon^{\mu\nu\alpha\beta}\langle x - {f_{+\mu\nu}, u_\alpha u_\beta}\rangle)</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>(i\epsilon^{\mu\nu\alpha\beta}\langle x - f_{+\mu\nu} f_{+\alpha\beta}\rangle)</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>(i\epsilon^{\mu\nu\alpha\beta}\langle x - f_{-\mu\nu} f_{-\alpha\beta}\rangle)</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>(i\epsilon^{\mu\nu\alpha\beta}\langle x + [f_{+\mu\nu}, f_{-\alpha\beta}]\rangle)</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>(\epsilon^{\mu\nu\alpha\beta}\langle f_{+\mu\nu}\rangle\langle x - u_\alpha u_\beta\rangle)</td>
<td>6</td>
<td>(-5NC)</td>
</tr>
<tr>
<td>(i\epsilon^{\mu\nu\alpha\beta}\langle f_{+\mu\nu}\rangle\langle f_{+\alpha\beta} x - \rangle )</td>
<td>7</td>
<td>(4NC)</td>
</tr>
<tr>
<td>(i\epsilon^{\mu\nu\alpha\beta}\langle f_{+\mu\nu}\rangle\langle f_{+\alpha\beta}\rangle\langle x - \rangle )</td>
<td>8</td>
<td>(-2NC)</td>
</tr>
<tr>
<td>(i\epsilon^{\mu\nu\alpha\beta}\langle f_{+\gamma\mu}\rangle\langle h_{\gamma\nu} u_\alpha u_\beta\rangle)</td>
<td>9</td>
<td>(2NC)</td>
</tr>
<tr>
<td>(i\epsilon^{\mu\nu\alpha\beta}\langle f_{+\gamma\mu}\rangle\langle f_{-\gamma\nu} u_\alpha u_\beta\rangle)</td>
<td>10</td>
<td>(-6NC)</td>
</tr>
<tr>
<td>(\epsilon^{\mu\nu\alpha\beta}\langle f_{+\mu\nu}\rangle\langle f_{+\gamma\alpha} h_{\gamma\beta}\rangle)</td>
<td>11</td>
<td>(4NC)</td>
</tr>
<tr>
<td>(\epsilon^{\mu\nu\alpha\beta}\langle f_{+\mu\nu}\rangle\langle f_{+\gamma\alpha} f_{-\gamma\beta}\rangle)</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>(\epsilon^{\mu\nu\alpha\beta}\langle \nabla_{\gamma} f_{+\mu\nu}\rangle\langle f_{+\nu\alpha} u_\beta\rangle)</td>
<td>13</td>
<td>(-4NC)</td>
</tr>
</tbody>
</table>

n.b. it depends on the form of \(\mathcal{L}_4\) [KK, Novotny 02], [Ananth.,Moussallam 02]
ChPT and resonances – introduction

Low energy effective theory of QCD at the next-to-next-to-leading order

\[ \mathcal{L}_\chi = \mathcal{L}_\chi^{(2)} + \mathcal{L}_\chi^{(4)} + \mathcal{L}_\chi^{(6)} \quad \rightarrow \text{free parameters: LECs} \]

For \( N_C \to \infty \) quantities can be calculated by means of tree graphs and:

\[ \mathcal{L}_\infty = \mathcal{L}_{GB}^{(2)} + \mathcal{L}_{GB}^{(4)} + \mathcal{L}_{GB}^{(6)} + \mathcal{L}_{\text{res}}^{(4)} + \mathcal{L}_{\text{res}}^{(6)} \]

(Chiral resonance Lagr.)

\[ \rightarrow \text{Integrating out resonance fields} \]

\[ \mathcal{L}_\chi = \mathcal{L}_{GB} + \mathcal{L}_{\chi, \text{res}} \]

(same form)

This can be used for a systematic estimates of LECs:

\( O(p^4) \): [Ecker, Gasser, Pich, de Rafael, ’89]
\( O(p^6) \): [Cirigliano, Ecker, Eidemuller, Kaiser, Pich & Portoles ’06],
odd sector: [K.K., Novotny ’11]
ChPT – dispersive approach

besides canonical ChPT one can use different approaches and combine them: two/three flavours, different counting, non-relativistic theories, use of analyticity and unitarity, etc...
dispersive approach based only on unitarity, analyticity (=why dispersive), crossing symmetry and chiral power-counting can serve as a model independent verification of ChPT.

**Reconstruction theorem**

Assuming validity of (subtracted) DR’s (and further conditions), we can reconstruct the amplitude of the process \( AB \rightarrow CD \):

\[
S(s, t; u) = R + \Phi_0(s) + [s(t - u) + (m_A^2 - m_B^2)(m_C^2 - m_D^2)]\Phi_1(s) + \text{crossed channels} + O(p^8),
\]

\( R \) - third order polynomial in \( s, t, u \) with same symmetries as \( S(s, t; u) \),

\[
\Phi_0(s) = 16s^3 \int_\Sigma^\infty \frac{dx}{x^3} \frac{\text{Im} f_0(x)}{x - s},
\]

\[
\Phi_1(s) = 48s^3 \int_\Sigma^\infty \frac{dx}{x^3} \frac{\text{Im} f_1(x)}{(x - s)\lambda_{AB}^{1/2}(x)\lambda_{CD}^{1/2}(x)},
\]

and similar for the \( t \)– and \( u \)– crossed channel \[ \lambda_{XY}(s) = (s - (m_X + m_Y)^2)(s - (m_X - m_Y)^2) \]

used e.g. for \( \eta \rightarrow 3\pi \), \( K \rightarrow 3\pi \) – see [KK, Knecht, Novotny, Zdrahal 09,11]
\( \pi^0 \) history

- **conception:** Yukawa '35, Kemmer '38
- **long birth:** Lewis, Oppenheimer, Wouthuysen '48, Carlson, Hooper, King '50, Bjorklund, Crandall, Moyer, York '50 “The existence of a neutral meson is clearly not required at the present stage of the experiments, but is the only one of the above five hypotheses which seems to fit the experimental data.”
- Steinberger, Panofsky, Steller '50: “It is clear from these properties that the gamma-rays are the decay products of neutral mesons.”
- Ekspong’97: “It was generally felt that the neutral pion marked the end for particle searches.”
- **two siblings:** \( \pi^+ \), \( \pi^- \), born: Lattes, Muirhead, Occhialini, Powell, ’47
\( \pi^0 \)'s properties

\[ I^G (J^{PC}) = 1^- (0^{-+}) \]

- mass \( m = 134.9766(6) \) MeV
- \( m_{\pi^\pm} - m_{\pi^0} = 4.5936(5) \) MeV
- mean life \( \tau = (8.4 \pm 0.6) \times 10^{-17} \) s
### $\pi^0$ Decay Modes

For decay limits to particles which are not established, see the appropriate Search sections (A$^0$ (axion) and Other Light Boson (X$^0$) Searches, etc.).

<table>
<thead>
<tr>
<th>Mode</th>
<th>Fraction ($\Gamma_i/\Gamma$)</th>
<th>Scale factor/Confidence level</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_1$</td>
<td>$2\gamma$</td>
<td>(98.823 ± 0.034) % S=1.5</td>
</tr>
<tr>
<td>$\Gamma_2$</td>
<td>$e^+ e^- \gamma$</td>
<td>(1.174 ± 0.035) % S=1.5</td>
</tr>
<tr>
<td>$\Gamma_3$</td>
<td>$\gamma$ positronium</td>
<td>($1.82 \pm 0.29$) × 10^{-9}</td>
</tr>
<tr>
<td>$\Gamma_4$</td>
<td>$e^+ e^+ e^- e^-$</td>
<td>($3.34 \pm 0.16$) × 10^{-5}</td>
</tr>
<tr>
<td>$\Gamma_5$</td>
<td>$e^+ e^-$</td>
<td>($6.46 \pm 0.33$) × 10^{-8}</td>
</tr>
<tr>
<td>$\Gamma_6$</td>
<td>$4\gamma$</td>
<td>$&lt; 2 \times 10^{-8}$ CL=90%</td>
</tr>
<tr>
<td>$\Gamma_7$</td>
<td>$\nu \bar{\nu}$</td>
<td>[a] $&lt; 2.7 \times 10^{-7}$ CL=90%</td>
</tr>
<tr>
<td>$\Gamma_8$</td>
<td>$\nu_e \bar{\nu}_e$</td>
<td>$&lt; 1.7 \times 10^{-6}$ CL=90%</td>
</tr>
<tr>
<td>$\Gamma_9$</td>
<td>$\nu_\mu \bar{\nu}_\mu$</td>
<td>$&lt; 1.6 \times 10^{-6}$ CL=90%</td>
</tr>
<tr>
<td>$\Gamma_{10}$</td>
<td>$\nu_\tau \bar{\nu}_\tau$</td>
<td>$&lt; 2.1 \times 10^{-6}$ CL=90%</td>
</tr>
<tr>
<td>$\Gamma_{11}$</td>
<td>$\gamma \nu \bar{\nu}$</td>
<td>$&lt; 6 \times 10^{-4}$ CL=90%</td>
</tr>
</tbody>
</table>

### Charge conjugation (C) or Lepton Family number (LF) violating modes

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</tr>
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<tbody>
<tr>
<td>$\Gamma_{12}$</td>
<td>$3\gamma$</td>
<td>$&lt; 3.1 \times 10^{-8}$ CL=90%</td>
</tr>
<tr>
<td>$\Gamma_{13}$</td>
<td>$\mu^+ e^-$</td>
<td>$&lt; 3.8 \times 10^{-10}$ CL=90%</td>
</tr>
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</table>
$\pi^0$ mean life, PDG history:

1985 $(8.4 \pm 0.6) \times 10^{-17}$ s

... 

2009 $(8.4 \pm 0.6) \times 10^{-17}$ s  
2010 $(8.4 \pm 0.5) \times 10^{-17}$ s  
2011 $(8.4 \pm 0.4) \times 10^{-17}$ s  
2012 $(8.52 \pm 0.18) \times 10^{-17}$ s ← PrimEx col.

... 

today $(8.52 \pm 0.18) \times 10^{-17}$ s

theory: $[KK, Moussallam]$ $(8.04 \pm 0.11) \times 10^{-17}$ s
$\pi^0$ life time

compare with $\eta$ decay:

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\( \pi^0 \rightarrow \gamma \gamma \)

- one of the most important processes for theory of particle physics
- \( \pi^0 \) lightest hadron ⇒ dominant decay mode \( \pi^0 \rightarrow \gamma \gamma \) (br=98.82%)
- non-existence of logarithmic correction to the current algebra result at NLO
- connection with the non-renormalization theorem ?
- new experimental activities
- theory – NNLO calculation: [KK,Moussallam’09] → see next
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- theory – NNLO calculation: [KK,Moussallam’09] \( \rightarrow \) see next
\[ \pi^0 \rightarrow \gamma \gamma: \text{ chiral expansion} \]

- in chiral limit exact due to QCD axial anomaly:

\[
\Gamma(\pi^0 \rightarrow \gamma \gamma) = \frac{m_{\pi^0}^3}{64\pi} \left( \frac{\alpha N_C}{3\pi F_\pi} \right)^2 = 7.73 \text{ eV}
\]

Correction to the current algebra prediction:
- using [Pagels and Zepeda '72] sum rules in [Kitazawa '85]
- NLO corrections are hidden in \( F \rightarrow F_{\pi^0} \) and \( O(p^6) \) LECs [Donoghue, Holstein, Lin '85] [Bijnens, Bramon, Cornet '88]
- in 3-flavour case we can study \( \pi^0, \eta, \eta' \) mixing, resulting to [Goity, Bernstein, Holstein '02]:

\[
\Gamma^{\text{NLO}} = 8.1 \pm 0.08 \text{ eV}
\]

in 2-flavour case EM corrections [Ananth., Moussallam '02]:

\[
\Gamma^{\text{NLO}} = 8.06 \pm 0.02 \pm 0.06 \text{ eV}
\]

- Quite recently another study based on dispersion relations, QCD sum rules, using only the value \( \Gamma(\eta \rightarrow \gamma \gamma) \) gives [Ioffe, Oganesian '07]:

\[
\Gamma^{\text{NLO}} = 7.93 \pm 0.11 \text{ eV}
\]
$\pi^0 \to \gamma\gamma$ at NNLO in 2 flavour ChPT: technical part

- NLO: a) One-loop diagrams with one vertex from $\mathcal{L}^{WZ}$, b) tree diagrams with one vertex from $\mathcal{L}^{WZ}$ and one vertex from $O(p^4)$ Lagrangian, c) tree diagrams with one vertex from $O(p^6)$ anomalous-parity sector

- $O(p^6)$ anomalous-parity sector from [Bijnens, Girlanda, Talavera '02]

- Representation of chiral field: $U = \sigma + i \frac{\pi \cdot \pi}{F}$, $\sigma = \sqrt{1 - \frac{\pi^2}{F^2}}$ (no $\gamma 4\pi$ vertex at LO)

- One-loop diagrams

- Two-loop diagrams

- Verification of $Z$-factor, $F_\pi / F$ [Bürgi '96], [Bijnens, Colangelo, Ecker, Gasser, Sainio '02]

- Double log checked by Weinberg consistency rel. [Colangelo '95]
\( \pi^0 \rightarrow \gamma\gamma \) at NNLO, result

\[
A_{\text{NNLO}} = \frac{e^2}{F_\pi} \left\{ \frac{1}{4\pi^2} \right. \\
+ \frac{16}{3} m_\pi^2 (-4c_3^{Wr} - 4c_7^{Wr} + c_{11}^{Wr}) + \frac{64}{9} B(m_d - m_u)(5c_3^{Wr} + c_7^{Wr} + 2c_8^{Wr}) \\
- \frac{M^4}{24\pi^2 F^4} \left( \frac{1}{16\pi^2} L_\pi \right)^2 + \frac{M^4}{16\pi^2 F^4} L_\pi \left[ \frac{3}{256\pi^4} + \frac{32F^2}{3} (2c_2^{Wr} + 4c_3^{Wr} + 2c_6^{Wr} + 4c_7^{Wr} - c_{11}^{Wr}) \right] \\
+ \frac{32M^2 B(m_d - m_u)}{48\pi^2 F^4} L_\pi [ -6c_2^{Wr} - 11c_3^{Wr} + 6c_4^{Wr} - 12c_5^{Wr} - c_7^{Wr} - 2c_8^{Wr} ] \\
+ \frac{M^4}{F^4} \lambda_+ + \frac{M^2 B(m_d - m_u)}{F^4} \lambda_- + \frac{B^2(m_d - m_u)^2}{F^4} \lambda_{--} \left\} 
\]

\[
\lambda_+ = \frac{1}{\pi^2} \left[ -\frac{2}{3} d_+^{Wr}(\mu) - 8c_6^r - \frac{1}{4}(l_4^r)^2 + \frac{1}{512\pi^4} \left( -\frac{983}{288} - \frac{4\zeta(3)}{3} + 3\sqrt{3} Cl_2(\pi/3) \right) \right] \\
+ \frac{16}{3} F^2 \left[ 8l_5^r(c_3^{Wr} + c_7^{Wr}) + l_4^r (-4c_3^{Wr} - 4c_7^{Wr} + c_{11}^{Wr}) \right] \\
\lambda_- = \frac{64}{9} \left[ d_-^{Wr}(\mu) + F^2 l_4^r (5c_3^{Wr} + c_7^{Wr} + 2c_8^{Wr}) \right] \\
\lambda_{--} = d_{--}^{Wr}(\mu) - 128F^2 l_7(c_3^{Wr} + c_7^{Wr}) .
\]

- **4 LECs** in 2 combinations of NLO
- **additional 4 LECs** in 3 combinations of NNLO

Is it at all possible to make some reliable prediction?
\[ \pi^0 \rightarrow \gamma\gamma: \text{modified counting} \]

- Use of \( SU(3) \) phenomenology via \( c_i^{Wr} \leftrightarrow C_i^{Wr} \) connection (based on [Gasser, Haefeli, Ivanov, Schmid '07,'08])

\[
c_i^{Wr} = \frac{\alpha_i}{m_s} + \left( \beta_i + \gamma_{ij} C_j^{Wr} + \delta_i \ln \frac{B_0 m_s}{\mu^2} \right) + O(m_s)
\]
\( \pi^0 \rightarrow \gamma\gamma: \) modified counting

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\[
c_i^{Wr} = \frac{\alpha_i}{m_s} + \left( \beta_i + \gamma_{ij} C_j^{Wr} + \delta_i \ln \frac{B_0 m_s}{\mu^2} \right) + O(m_s)
\]

- implementation of modified counting

\[
m_u, m_d \sim O(p^2) \quad \text{and} \quad m_s \sim O(p)
\]

Result:

\[
A_{NNLO}^{mod} = \frac{e^2}{F_\pi^2} \left\{ \frac{1}{4\pi^2} - \frac{64}{3} m_\pi^2 C_7^{Wr} + \frac{1}{16\pi^2} \frac{m_d - m_u}{m_s} \left[ 1 - \frac{3}{2} \frac{m_\pi^2}{16\pi^2 F_\pi^2 L_\pi} \right] \right. \\
+ 32B(m_d - m_u) \left[ \frac{4}{3} C_7^{Wr} + 4C_8^{Wr} \left( 1 - 3 \frac{m_\pi^2}{16\pi^2 F_\pi^2 L_\pi} \right) \right] \\
- \left. \frac{1}{16\pi^2 F_\pi^2} \left( 3L_7^r + L_8^r - \frac{1}{512\pi^2}(L_K + \frac{2}{3} L_\eta) \right) \right\} \\
- \frac{1}{24\pi^2} \left( \frac{m_\pi^2}{16\pi^2 F_\pi^2 L_\pi} \right)^2
\]
Phenomenology

- $F_{\pi}$ from [Marciano, Sirlin '93] $\pi l^2$ decay:

$$\Gamma = \frac{G_F^2}{4\pi} |V_{ud}|^2 m_l^2 m_\pi (1 - z^2)^2 \left(1 + \frac{2\alpha}{\pi} \ln \frac{m_Z}{m_\rho}\right)(1 + C_1 + \ldots)$$

- $C_1$ estimate via ChPT
  - [Knecht, Neufeld, Rupertsberger, Talavera '00]

$$C_1 = \frac{Z}{4} \left(3 + 2 \ln \frac{m_\pi^2}{m_\rho^2} + \ln \frac{m_K^2}{m_\rho^2}\right) - \frac{1}{2} + f(K_i^r, X_i^r)$$

  - [Ananth., Moussallam '02] [Descotes, Moussallam '06]

$$C_1 = -2.56 \pm 0.50$$

- $V_{ud}$ [Towner, Hardy '08]

$$V_{ud} = 0.97418(26)$$

$$\Rightarrow F_{\pi} = 92.22 \pm 0.07 \text{ MeV}$$

[rem.: of course if SM is correct – see page 37]
\( \pi \rightarrow \gamma \gamma: \) Phenomenology

- \( F_\pi = 92.22 \pm 0.07 \) MeV

using quark mass ratio (from lattice), pseudo-scalar meson masses, \( R \) from \( \eta \rightarrow 3\pi \) (ChPT: [Bijnens,Ghorbani ’07])

- \( \frac{m_d - m_u}{m_s} = (2.29 \pm 0.23) \times 10^{-2} \)

- \( B(m_d - m_u) = (0.32 \pm 0.03) \, M_{\pi^0}^2 \)

- \( 3L_7 + L_8^r(\mu) = (0.10 \pm 0.06) \times 10^{-3} \) \((\mu = M_\eta)\) (from pseudo-scalar meson masses formula [Gasser, Leutwyler ’85])

- \( C_7^W = 0 \) (more precisely \( C_7^W \ll C_8^W \), motivated by simple resonance saturation)

- \( C_8^W = (0.58 \pm 0.2) \times 10^{-3} \text{GeV}^{-2} \) (from \( \eta \rightarrow 2\gamma \))

result

\[ \Gamma_{\pi^0 \rightarrow 2\gamma} = (8.09 \pm 0.11) \text{eV} \]

[or \( \tau_{\pi^0} = (8.04 \pm 0.11) \times 10^{-17} \text{s} \)]
\( \pi^0 \rightarrow \gamma \gamma: \text{leading logs} \)  
(for details see [Bijnens, KK, Lanz'12])

Leading logarithm contribution of individual orders in percent of the leading order:

Adler-Lee-Treiman-Zee-Terentev theorem on triangle and box anomaly

\[
F^{3\pi}(0, 0, 0) = \frac{1}{eF^2_{\pi}} F_{\pi\gamma\gamma}(0, 0)
\]

is valid up to 2-loop order for LL beyond the soft-photon limit
\[
\pi^0 \rightarrow e^+ e^-
\]

- first studied by [S. Drell ’59]
- radiative corrections: [L. Bergström ’83]
- most recent experiment: KTeV E799-II [Abouzaid’07]
- radiative corrections play important role
- new preliminary experimental studies
\[ \pi^0 \rightarrow e^+ e^- \]

KTeV’s measurement:

\[
\frac{\Gamma(\pi^0 \rightarrow e^+ e^-, x > 0.95)}{\Gamma(\pi^0 \rightarrow e^+ e^-\gamma, x > 0.232)} = (1.685 \pm 0.064 \pm 0.027) \times 10^{-4}.
\]

by extrapolating the Dalitz branching ratio to the full range of \( x \)

\[
B(\pi^0 \rightarrow e^+ e^- (\gamma), x > 0.95) = (6.44 \pm 0.25 \pm 0.22) \times 10^{-8}.
\]

Extrapolating the radiative tail using Bergström:

\[
B_{KTeV}^{\text{no-rad}}(\pi^0 \rightarrow e^+ e^-) = (7.48 \pm 0.29 \pm 0.25) \times 10^{-8}.
\]

Theoretical prediction [Dorokhov, Ivanov ’07, ’10]

\[
B_{\text{SM}}^{\text{no-rad}}(\pi^0 \rightarrow e^+ e^-) = (6.23 \pm 0.09) \times 10^{-8}. \tag{2}
\]

3.3 \( \sigma \) \( \Rightarrow \) New physics?

In any case, radiative corrections play an important role in the analysis
\[ \pi^0 \rightarrow e^+ e^- \]

Radiative corrections $\rightarrow$ two-loop graphs

(a) (b)
(c) (d)
(e) (f)
\[ \pi^0 \rightarrow e^+ e^- \]

- two-loop contributions, together with Bremsstrahlung (= Dalitz) [Dorokhov et al. '08], [Vasko, Novotny '11], [Husek, KK, Novotny'14]
- counter-term chiral Lagrangian for \( \pi^0 \bar{\ell} \ell \) [Savage et al’92]
- modelled using the resonances [Knecht ’99]

\[ \chi^{(r)}_{\text{LMD}}(M_\rho) = 2.2 \pm 0.9 \]

- rem.: different models possible, see e.g. [Masjuan, Sanchez-Puertas ‘15], for \( \chi = 2.76(23) \)
- KTeV implies [Husek, KK, Novotny’14]

\[ \chi^{(r)}_{\text{KTeV}}(M_\rho) = 4.5 \pm 1.0 \]

- original discrepancy down to 2 \( \sigma \) level
- note: weak contributions mediated via \( \pi^0 \rightarrow Z^* \rightarrow e^+ e^- \) three orders of magnitude smaller than EM
**Dalitz decay**

[K.K., Knecht, Novotný '06]

**History**
- First calculated by [Dalitz '51].
- Radiative corrections studied by [Joseph '60], [Lautrup, Smith'71], [Mikaelian, Smith'72] and during the 1980s by Tupper, Grose, Samuel, Lambin, Pestieau, Roberts...

\[
\begin{align*}
\pi^0(P) &\rightarrow q e^+(p_+) e^-(p_-) \\
\gamma(k) &
\end{align*}
\]

\[
x = \frac{m_{ee}^2}{M_{\pi}^2}, \quad y = \frac{E_+ - E_-}{E_{\gamma}} \bigg|_{\pi^0 \rightarrow 0}
\]

NLO studied via \( \delta(x, y) \) and \( \delta(x) \):

\[
\frac{d\Gamma}{dx\,dy} = \delta(x, y) \frac{d\Gamma^{LO}}{dx\,dy}, \quad \frac{d\Gamma}{dx} = \delta(x) \frac{d\Gamma^{LO}}{dx}.
\]

with (point-like pion)

\[
\begin{align*}
\frac{d\Gamma^{LO}}{dx\,dy} &= \frac{\alpha^3}{(4\pi)^4} \frac{M_{\pi^0}^2}{F_{\pi}^2} \frac{(1-x)^3}{x^2} \left[M_{\pi^0}^2 x(1+y^2) + 4m^2\right], \\
\frac{d\Gamma^{LO}}{dx} &= \frac{\alpha^3}{(4\pi)^4} \frac{8M_{\pi^0}}{3} \frac{(1-x)^3}{x^2} (xM_{\pi^0}^2 + 2m^2).
\end{align*}
\]
Dalitz decay: Anatomy of the amplitude

- one-photon reducible graphs: electron-positron pair is produced by a single photon (Dalitz pair)

- one-photon irreducible graphs

LO, order $O(e^5)$
Dalitz decay: slope parameter

\[ \Gamma_\mu^1\gamma^R(p_+, p_-, k) = ie^2 \varepsilon_\mu^{\nu\alpha\beta} q_\alpha k_\beta F_{\pi^0\gamma\gamma^*}(q^2) iD^{T\mu}_{\nu\rho}(q) (-ie) \Lambda^\rho \]

\( F_{\pi^0\gamma\gamma^*}(q^2) \) is related to the doubly off-shell form factor \( A_{\pi^0\gamma\gamma^*}(q_1^2, q_2^2) \)

\[ \int d^4x e^{il\cdot x} \langle 0| T(j^\mu(x)j^\nu(0)| \pi^0(P) \rangle = -i\varepsilon^{\mu\nu\alpha\beta} l_\alpha P_\beta A_{\pi^0\gamma\gamma^*}(l^2, (P-l)^2) \]

One can define a slope parameter \( a_{\pi} \)

\[ F_{\pi^0\gamma\gamma^*}(q^2) = F_{\pi^0\gamma\gamma^*}(0) \left[ 1 + a_{\pi} \frac{q^2}{M_{\pi^0}^2} + \cdots \right], \]

\[ \frac{d\Gamma^{exp}}{dx} - \delta_{QED}(x) \frac{d\Gamma^{LO}}{dx} = \frac{d\Gamma^{LO}}{dx} \left[ 1 + 2x a_{\pi} \right]. \]
Our works provide a detailed analysis of NLO radiative corrections to the Dalitz decay amplitude.

The off-shell pion-photon transition form factor was included: this requires a treatment of non perturbative strong interaction effects.

The one-photon irreducible contributions, which had been usually neglected, were included. We have shown that, although these contributions are negligible as far as the corrections to the total decay rate are concerned, they are however sizeable in regions of the Dalitz plot which are relevant for the determination of the slope parameter $a_\pi$ of the pion-photon transition form factor.

Our prediction for the slope parameter $a_\pi = 0.029 \pm 0.005$ is in good agreement with the determinations obtained from the (model dependent) extrapolation of the CELLO and CLEO data. Unfortunately, the experimental error bars on the latest values of $a_\pi$ extracted from the Dalitz decay are still too large.

used in NA48 analysis for the search of dark photon [1504.00607]
Dalitz decay: one last remark

- important for normalization in rare pion and kaon decays
- PDG prediction was stable for 30y (LAMPF ’81).
- reason for the change: ALEPH ’08 (in principle also KTeV).
- Theoretical prediction is stable and well-understood
- shift in the central value of Dalitz would have an impact in other measurements, eg.

\[ K_L \rightarrow e^+ e^- \gamma = (9.13 \pm 0.26) \times 10^{-6} \left|_{\text{KTeV+pdg}} \right. \rightarrow (8.70 \pm 0.13) \times 10^{-6} \left|_{\text{KTeV+KTeV}} \right. \]
Double Dalitz decay

History

- Determination of parity of pion via $\pi^0 \rightarrow \gamma\gamma$ [Yang '50], experimentally difficult
- Using internal conversion [Kroll, Wada '55]
- First measurement (and today's PDG number) [Samios et al. '62], hydrogen bubble chamber: $8 \times 10^6 \pi^0$-decays on approx 800 thousand pictures $\rightarrow$ 200 double-dalitzs (10t. dalitzs)

\[
B(\pi^0 \rightarrow e^+e^-e^+e^-) = (3.18 \pm 0.3) \times 10^{-5}
\]
\[
\pi^0 \text{ is pseudoscalar (only } 3.6\sigma \text{ significance)}
\]

- [Miyazaki and E. Takasugi '73] adding the effect of lepton exchange to Kroll-Wada
- New study: [Barker et al. '03] (some disagreement with previous)
- New measurement: KTeV '08
  - Confirmation of negative $\pi^0$ parity
  - First searches for parity & CPT violation
  - \[
  B(\pi^0 \rightarrow e^+e^-e^+e^-) = (3.46 \pm 0.19) \times 10^{-5}
  \]
Double Dalitz decay collaboration with M. Knecht, J. Novotný

It seems natural to convert the on-shell photon to the other Dalitz pair and obtain immediately Double Dalitz decay. This is true for LO:

\[ \pi^0 \rightarrow e^+ e^- \gamma. \]

However, for higher orders we have new topologies [Barker et al. '03]:

We are recalculating these results and try to put them together with our parameters introduced in the context of \( \pi^0 \rightarrow e^+ e^- \gamma \).
\[ \pi^0 \rightarrow Ps \gamma \]

- decay of pion to **positronium** and photon, very similar to the Dalitz decay (same particle content)
- Due to binding energy, threshold for \( Ps \) is below \( e^+e^- \)
- QED+non relativistic bound state formalism Nemenov '72:

\[
\frac{\Gamma(\pi^0 \rightarrow Ps + \gamma)}{\Gamma(\pi^0 \rightarrow \gamma\gamma)} = \frac{\alpha^4}{2} \sum_{n=1}^{\infty} \frac{1}{n^3} \left(1 + O\left(\frac{M_{Ps}^2}{m_{\pi}^2}, \alpha\right)\right) \approx \frac{\alpha^4}{2} \zeta(3) \approx 1.7 \times 10^{-9}
\]

(the sum over all radial excitations of the \( S \)-wave \( J = 1 \) positronium state)
- radiative correction calculated and small Vysotskii '79
- experiment Serpukhov '89 (=pdg)

\[
\rho_\pi = (1.84 \pm 0.29) \times 10^{-9}
\]
\[ \pi^0 \rightarrow 4\gamma \]

- important background for \( \pi^0 \rightarrow 3\gamma \)
- interesting probe of the light-by-light scattering

\[ \text{theoretical estimate: Bratkovskaya, Kuraev, Silagadze '95} \]

\[ \text{Br}(\pi^0 \rightarrow 4\gamma) = (2.6 \pm 0.1) \times 10^{-11} \]

- 3 orders below experimental limit, LAMPF '88

\[ \text{Br}_{pdg}(\pi^0 \rightarrow 4\gamma) < 2 \times 10^{-8} \]
\[ \pi^0 \rightarrow \nu \bar{\nu} \]

- strictly speaking, from the beginning must be discussed beyond SM
- theoretical calculation, see EW contribution to \( \pi^0 \rightarrow e^+ e^- \)

\[ A = \sqrt{2} G_F F_{\pi} m_\nu \bar{u} \gamma_5 v \]

\[ \Rightarrow \quad \text{Br}_{\nu \bar{\nu}} = \left( \frac{4\pi F_{\pi}^2 G_F}{\alpha} \right)^2 \left( \frac{m_\nu}{M_\pi} \right)^2 \sqrt{1 - \frac{4m_\nu^2}{M_\pi^2}} \]

- note the maximum for the ratio \( m_\nu = M_\pi / \sqrt{6} \)
- pdg limit on (tau) \( m_\nu < 18.2 \text{ MeV} \) leads to \( \text{Br}_{\nu \bar{\nu}} < 5 \times 10^{-10} \)
- cosmology limits push it much lower
- exp.limit (E949/pdg): \( \sim 10^{-7} \)
- possible impact of \( \pi^0 \rightarrow \nu \bar{\nu}(\gamma) \) in stellar cooling processes of the type \( \gamma \gamma \rightarrow \nu \bar{\nu}(\gamma) \)
\( \pi^0 \rightarrow \nu \bar{\nu} \gamma \)

- “Wolfram” process [Wolfram’76], see however [Arnelos, Marciano, Parsa’82]
  - without helicity supression
  - represents weak radiative decay

- \( \text{Br}_{\nu \bar{\nu} \gamma} \approx 10^{-18} \)
- possible test of \( \tau \) neutrino magnetic moment [Grasso, Lusignoli’92]
- exp. limit (E787) on \( \pi^0 \rightarrow \gamma + \text{“nothing”} \) \( \text{Br} \lesssim 10^{-4} \)
\( \pi^0 \) and new physics

- one crucial ingredient for chiral dynamics: \( F_\pi \)
- \( F_\pi \) from \( \pi l_2 \) based on SM; deviation from standard \( V - A \) leads to an effective \( \hat{F}_\pi \) [Bernard,Oertel,Passemar,Stern '08]

\[
F_\pi^2 = \hat{F}_\pi^2 (1 + \epsilon), \quad \text{with} \quad \epsilon \sim V_{ud}^R / V_{ud}^L
\]

- connection between \( F_\pi \) and \( F_{\pi^0} \) tiny [KK,Moussallam '09]

\[
\left. \frac{F_{\pi^+}}{F_{\pi^0}} \right|_{QCD} - 1 = \frac{B^2(m_d - m_u)^2}{F_\pi^4} \left[ -16 c_9^r(\mu) - \frac{l_7}{16\pi^2} \left( 1 + \log \frac{m_\pi^2}{\mu^2} \right) \right]
\]

\[
\simeq 0.7 \times 10^{-4}.
\]

\[ \Rightarrow \text{one can thus use} \quad \pi^0 \rightarrow \gamma\gamma \text{ for determination of} \quad F_\pi: \]

\[
F_\pi \simeq F_{\pi^0} = 93.85 \pm 1.3(\text{exp.}) \pm 0.6(\text{theory}) \text{ MeV} = 93.85 \pm 1.4 \text{ MeV}
\]

- n.b. \( \hat{F}_\pi = 92.22(7) \Rightarrow \epsilon \approx 3 - 4\% \text{ 1}\sigma \text{ significance for right-handed currents} \)
Summary

- brief introduction to low energy effective field theory
- presented short overview of all allowed $\pi^0$ decays, namely
  - $\pi^0 \rightarrow \gamma\gamma$
  - $\pi^0 \rightarrow e^+e^-$
  - $\pi^0 \rightarrow e^+e^-\gamma$ (Dalitz decay)
  - $\pi^0 \rightarrow e^+e^-e^+e^-$ (double Dalitz)
  - $\pi^0 \rightarrow \textit{posit.}\gamma$
  - $\pi^0 \rightarrow 4\gamma$
  - $\pi^0 \rightarrow \nu\bar{\nu}(\gamma)$
- possible new physics effects
- important for phenomenology but also for studying basic and conceptual theoretical properties
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Thank you.