The $\Lambda(1405)$ and new nonconventional baryon states

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Historical overview
The two states of the $\Lambda(1405)$
Reactions with $\Lambda(1405)$ production
New reactions from $\Lambda_b$ and $\Lambda_c$ decays
Recent pentaquark discovery
Suggesting new reactions to observe hidden charm states with strangeness
A bit of history

The Λ(1405) has always been the “odd” state in quark models. Even now:

J.~Ferretti, R.~Bijker, G.~Galatà, H.~García-Tecocoatzi and E.~Santopinto,  

The Λ(1405) is displaced 200 MeV and has a πΣ width 5 times larger than experiment.

Early in 1959 Dalitz and collaborators predicted this resonance as a 
Kbar N bound state, coupling to an open channel πΣ.

Real progress appeared with the advent of chiral unitary theory: Combination of 
Chiral Lagrangians and the use of the Lippmann Schwinger Equation 
coupled channels to Kbar N and form factors to regularize the loops.

A step forward was given in E. O. , A. Ramos in NPA (1998)
Chiral Lagrangian for pseudoscalar-baryon interaction

At lowest order in momentum the interaction Lagrangian reduces to

\[ L_{1}^{(B)} = \langle \bar{B} i \gamma^{\mu} \frac{1}{4 f^{2}} [ (\Phi \partial_{\mu} \Phi - \partial_{\mu} \Phi \Phi) B - B (\Phi \partial_{\mu} \Phi - \partial_{\mu} \Phi \Phi) ] \rangle \]

for \( K^- p \) scattering, there are 10 channels, namely \( K^- p, \bar{K}^0 n, \pi^0 \Lambda, \pi^0 \Sigma^0, \pi^+ \Sigma^-, \pi^- \Sigma^+, \eta \Lambda, \eta \Sigma^0, K^+ \Xi^- \) and \( K^0 \Xi^0 \). In the case of \( K^- n \) scattering the coupled channels are: \( K^- n, \pi^0 \Sigma^-, \pi^- \Sigma^0, \pi^- \Lambda, \eta \Sigma^- \) and \( K^0 \Xi^- \). These amplitudes have the form

\[ V_{ij} = -C_{ij} \frac{1}{4 f^{2}} (k^0_{j} + k^0_{i}) \]

One solves the Bethe- Salpeter equation in coupled channels

\[ T = (1 - VG)^{-1} V \]

This produces transition amplitudes from \( K^- p \) to any other channel. Gives rise to resonances, \( \Lambda(1405), \Lambda(1670), \Sigma(1650) \)…
\[ \Phi = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & K^0 \\ K^- & -\frac{1}{\sqrt{2}} \eta_8 & -\frac{1}{\sqrt{6}} \eta_8 \end{pmatrix} \]

\[ B = \begin{pmatrix} \frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & n \\ \Xi^- & \Xi^0 & -\frac{1}{\sqrt{6}} \Lambda \end{pmatrix} \]

\[ G_I = i 2 M_I \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(P - q)^2 - M_I^2 + i\varepsilon} \frac{1}{q^2 - m_i^2 + i\varepsilon} \]

\[ = \frac{2 M_I}{16\pi^2} \left\{ a_I(\mu) + \ln \frac{M_I^2}{\mu^2} + \frac{m_i^2 - M_I^2}{2s} \ln \frac{m_i^2}{M_I^2} + \frac{\bar{q}_I}{\sqrt{s}} \left[ \ln(s - (M_I^2 - m_i^2) + 2\bar{q}_I \sqrt{s}) + \ln(s + (M_I^2 - m_i^2) + 2\bar{q}_I \sqrt{s}) \right. \right. \]

\[ - \ln(-s + (M_I^2 - m_i^2) + 2\bar{q}_I \sqrt{s}) - \ln(-s - (M_I^2 - m_i^2) + 2\bar{q}_I \sqrt{s}) \]
The off shell terms go into renormalization of the tree level.

\[ t_{ij} = V_{ij} + V_{il} G_l T_{lj} \]

\[ V_{il} G_l T_{lj} = i \int \frac{d^4q}{(2\pi)^4} \frac{M_l}{E_l(q)} \frac{V_{il}(k, q) T_{lj}(q, k')}{k^0 + p^0 - q^0 - E_l(q) + i\epsilon} \frac{1}{q^2 - m_l^2 + i\epsilon} \]

\[ V_{off}^2 = C(k^0 + q^0)^2 = C(2k^0 + q^0 - k^0)^2 \]

\[ = C^2(2k^0)^2 + 2C(2k^0)(q^0 - k^0) + C^2(q^0 - k^0)^2 \]

\[ 2iV_{on} \int \frac{d^3q}{(2\pi)^3} \int \frac{dq^0}{2\pi} \frac{M}{E(q)} \frac{q^0 - k^0}{k^0 - q^0} \frac{1}{q^{02} - \omega(q)^2 + i\epsilon} = \]

\[ = -2V_{on} \int \frac{d^3q}{(2\pi)^3} \frac{M}{E(q)} \frac{1}{2\omega(q)} \sim V_{on} q_{max}^2 \]

The off shell terms go into renormalization of the tree level.

Factorization on shell allows V and T to be taken out of the integral → The remaining integral is done analytically and the L. S. eqn. becomes an algebraic one.

A formal and general derivation, using dispersion relations is done in J.A. Oller and U.G. Meissner, PLB 500 (2001)

Since then, every one uses the “on shell” factorization
Coupled channels:

\( K^- p, \bar{K}^0 n, \pi^0 \Lambda, \pi^0 \Sigma^0, \pi^+ \Sigma^-, \pi^- \Sigma^+, \eta \Lambda, \eta \Sigma^0, K^0 \Xi^0 \) and \( K^+ \Xi^- \)


The two poles were first noted in Oller and Meissner PLB (2001) paper
The two poles are now obtained by all groups using the chiral unitary approach.

Many other resonances are also dynamically generated from the meson baryon interaction. For later reference:

\( N^*(1535), N^*(1650) \) \( 1/2^- \) couple strongly to \( K \Lambda \) and \( K \Sigma \)

They can be considered **Hidden Strangeness States**

See also recent review on the PDG by Hyodo and Meissner, making the two pole structure official.

**Important thing** about the \( \Lambda(1405) \) and other resonances dynamically generated from the meson-baryon interaction:

In reactions where the resonances are produced they are not produced directly:
The components are produced and when they interact the resonance is produced.

This mean: great prediction power

bigger risk of making wrong predictions: But this is not the case so far.
Weinberg-Tomozawa

Minimal substitution from Weinberg-Tomozawa

$\gamma p \rightarrow K^+ \Lambda(1405)$

Meson baryon interaction in coupled channels generate the $\Lambda(1405)$

Nacher, E. O. Toki and Ramos
PLB (1999)

Improved model: S. X. Nakamura and D. Jido, PTEP (2014)
The $K^- p \rightarrow \pi^0 \pi^0 \Sigma^0$ reaction selects mostly narrow pole.

The $\pi^- p \rightarrow K^0 \pi \Sigma$ reaction selects mostly wide pole.
Kaonic production of $\Lambda(1405)$ off deuteron target in chiral dynamics

D. Jido$^1$, E. Oset$^2$, and T. Sekihara$^3$

PRC 2010

Recent work of Miyagawa, Haidenbauer question this approach arXiv:1202.4272
But it neglects the potential energy of the nucleons.

New results with Watson formalism or spectral functions support Jido’s results

Fig. 2. Diagrams for the calculation of the $K^-d \rightarrow \pi\Sigma n$ reaction. $T_1$ and $T_2$ denote the scattering amplitudes for $KN \rightarrow \bar{K}N$ and $KN \rightarrow \pi\Sigma$, respectively.
Taulogical statement: to learn about the $\Lambda(1405)$ the best way is to see reactions where it is produced!

However, most of the work of different groups is based on fits to scattering data, or threshold reactions.

Message: production reactions should be fitted at the same time!
A. Cieply, M. Mai, U.G. Meissner and J. Smejkal, ArXiv 1603.0253
Description of the $\gamma p \rightarrow K^+ \pi\Sigma$ reaction

L. Roca and EO, PRC2013

We look first at $\pi^0\Sigma^0$ which has $I=0$.

$$t(W) = b(W)G_{\pi\Sigma}T_{\pi\Sigma,\pi\Sigma}^{I=0} + c(W)G_{\bar{K}N}T_{\bar{K}N,\pi\Sigma}^{I=0}$$

Graph showing $|T|^2$ vs. $\sqrt{s}$ (MeV) with different curves for $T_{\bar{K}N,\pi\Sigma}^{I=0}$ and $T_{\pi\Sigma,\pi\Sigma}^{I=0}$.

$|T|^2$ (MeV$^2$)

- $T_{\bar{K}N,\pi\Sigma}^{I=0}$
- $T_{\pi\Sigma,\pi\Sigma}^{I=0}$

$\sqrt{s}$ (MeV)

1300 to 1500
Results fitting the coefficients $b(W)$ and $C(W)$ to data of Moriya et. al on $\pi^0\Sigma^0$, PRC 2013
Improved fit
\[
C_{ij} = \begin{pmatrix}
3\alpha_1 & -\sqrt{\frac{3}{2}}\alpha_2 \\
-\sqrt{\frac{3}{2}}\alpha_2 & 4\alpha_3
\end{pmatrix}
\]

We allow the regulators $a_i$ to vary a bit with respect to the original values.

We respect unitarity in coupled channels. $b(W), c(W)$ are complex numbers for each energy, $\alpha_i$ are parameters of the potential.

\[
a_{KN} \rightarrow \alpha_4 a_{KN}, \quad a_{\pi \Sigma} \rightarrow \alpha_5 a_{\pi \Sigma}
\]
$\pi^0\Sigma^0$ are only in $l=0$

Solution 1 is favored
Global fit to all photoproduction data

\[ C_{ij}^1 = \begin{pmatrix} 1 & -1 & -\sqrt{\frac{3}{2}} \\ -1 & 2 & 0 \\ -\sqrt{\frac{3}{2}} & 0 & 0 \end{pmatrix} \]

the order of the channels is \( \bar{K}N, \pi \Sigma, \) and \( \pi \Lambda \)

\[ |\pi^0 \Sigma^0\rangle = \sqrt{\frac{2}{3}} |20\rangle - \frac{1}{\sqrt{3}} |00\rangle, \]

\[ |\pi^+ \Sigma^-\rangle = -\frac{1}{\sqrt{6}} |20\rangle - \frac{1}{\sqrt{2}} |10\rangle - \frac{1}{\sqrt{3}} |00\rangle, \]

\[ |\pi^- \Sigma^+\rangle = -\frac{1}{\sqrt{6}} |20\rangle + \frac{1}{\sqrt{2}} |10\rangle - \frac{1}{\sqrt{3}} |00\rangle. \]

\[ t_{\gamma p \to K^+ \pi^0 \Sigma^0}(W) = b_0(W)G_{\pi \Sigma}^{I=0}T_{\pi \Sigma, \pi \Sigma}^{I=0} + c_0(W)G_{\bar{K}N}^{I=0}T_{\bar{K}N, \pi \Sigma}^{I=0}, \]

\[ t_{\gamma p \to K^+ \pi^\pm \Sigma^\mp}(W) = b_0(W)G_{\pi \Sigma}^{I=0}T_{\pi \Sigma, \pi \Sigma}^{I=0} + c_0(W)G_{\bar{K}N}^{I=0}T_{\bar{K}N, \pi \Sigma}^{I=0} + \sqrt{\frac{3}{2}}(b_1(W)G_{\pi \Sigma}^{I=1}T_{\pi \Sigma, \pi \Sigma}^{I=1} + c_1(W)G_{\bar{K}N}^{I=1}T_{\bar{K}N, \pi \Sigma}^{I=1} + d_1(W)G_{\pi \Lambda}^{I=1}T_{\pi \Lambda, \pi \Sigma}^{I=1}), \]
FIG. 2. (Color online) Fit to photoproduction data with fixed unitary amplitudes of $\alpha_i = 1$ and $\beta_i = 1$. Red: $\pi^0\Sigma^0$; blue: $\pi^-\Sigma^+$, green $\pi^+\Sigma^-$. Experimental data are from Ref. [2].
\[ C_{ij}^{0} = \begin{pmatrix} 3\alpha_{11}^{0} & -\sqrt{\frac{3}{2}}\alpha_{12}^{0} \\ -\sqrt{\frac{3}{2}}\alpha_{12}^{0} & 4\alpha_{22}^{0} \end{pmatrix} \]

for isospin \( I = 0 \) and

\[ C_{ij}^{1} = \begin{pmatrix} \alpha_{11}^{1} & -\alpha_{12}^{1} & -\sqrt{\frac{3}{2}}\alpha_{13}^{1} \\ -\alpha_{12}^{1} & 2\alpha_{22}^{1} & 0 \\ -\sqrt{\frac{3}{2}}\alpha_{13}^{1} & 0 & 0 \end{pmatrix} \]

for isospin \( I = 1 \).

\( \beta_i \) are the coefficients multiplying the standard subtraction contents.

<table>
<thead>
<tr>
<th>( \alpha_{11}^{0} )</th>
<th>( \alpha_{12}^{0} )</th>
<th>( \alpha_{22}^{0} )</th>
<th>( \alpha_{11}^{1} )</th>
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<th>( \beta_{2} )</th>
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<td>1.037</td>
<td>1.466</td>
<td>1.668</td>
<td>0.85</td>
<td>0.93</td>
<td>1.056</td>
<td>0.77</td>
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<th>( I = 0 )</th>
<th>( I = 1 )</th>
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<td>Poles</td>
<td>1352 − 48i</td>
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<td>(</td>
<td>g_{KN}</td>
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<tr>
<td>(</td>
<td>g_{\pi\Sigma}</td>
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No poles for $l=1$ are found, but amplitudes resemble much the shape of the $a_0(980)$ “resonance”.

The $l=1$ amplitude is only $1/4$ of that of $l=0$, and essential to describe the photoproduction data.

Claim of $l=1$ state from analysis of $k^- p \rightarrow \Lambda \pi^+ \pi^-$. 

\[ \frac{d\sigma_{\pi\eta}}{dE_{cm}} \ (\mu b/GeV) \]
Other methods to know that the $\Lambda(1405)$ is of molecular type: Use Weinberg compostiness condition


K. Miyahara and T. Hyodo, ArXiv 1512.02735


The method works very well for bound states. For resonances the interpretation is more subtle, but, even then, all studies conclude the molecular nature of the two $\Lambda(1405)$ states
What is next:

new reactions to produce the $\Lambda(1405)$
Predictions for the $\Lambda_b \rightarrow J/\psi \, \Lambda(1405)$ decay

L. Roca, M. Mai, E.Oset and U.G. Meissner, EPJC 2015

$|H\rangle = |K^-p\rangle + |K^0n\rangle - \frac{\sqrt{2}}{3} |\eta\Lambda\rangle + \frac{2}{3} |\eta'\Lambda\rangle$

$u\,d$ quarks in I=0

$u\,d$ quarks in I=0 (spectators) an s quark $\rightarrow$ total I=0

$\mathcal{M}_j(M_{\text{inv}}) = V_p \left( h_j + \sum_i h_i G_i(M_{\text{inv}}) t_{ij}(M_{\text{inv}}) \right)$

$h_{\pi^0\Sigma^0} = h_{\pi^+\Sigma^-} = h_{\pi^-\Sigma^+} = 0, \; h_{\eta\Lambda} = -\frac{\sqrt{2}}{3}$

$h_{K^-p} = h_{K^0n} = 1, \; h_{K^0\Xi^-} = h_{K^0\Xi^0} = 0$
Weak decay of $\Lambda_c^+$ for the study of $\Lambda(1405)$ and $\Lambda(1670)$

There are technical difficulties at Belle to extract the $\pi\Sigma$ channel: John Yelton (in parallel session)
It would be interesting to compare with the results of M. Hussain Monday parallel session.
The pentaquark era:
We have there $J/\psi K^- p$, the final state in the LHCb pentaquark experiment.

Note the large deviation from phase space for $K^- p$.
While for $J/\psi p$ one has essentially phase space except for the peak.

Large concentration of strength around threshold.
How can the peak in $J/\psi$ appear? The $J/\psi$ N interaction is very weak!!
Predictions for hidden charm Baryon states

\[
\begin{array}{cccc}
(I, S) & z_R & g_a \\
(1/2, 0) & \bar{D}^* \Sigma_c & \bar{D}^* \Lambda_c^+ & J/\psi N \\
& 4415 - 9.5i & 2.83 - 0.19i & -0.07 + 0.05i & -0.85 + 0.02i \\
& \multicolumn{1}{c}{2.83} & \multicolumn{1}{c}{0.08} & \multicolumn{1}{c}{0.85} \\
\end{array}
\]

C W Xiao, J Nieves, E. O, PRD 2013: \( \bar{D}^* \text{bar } \Sigma_c^* \) channel included

\[
\begin{array}{cccccc}
4417.04 + i4.11 & J/\psi N & \bar{D}^* \Lambda_c & \bar{D}^* \Sigma_c & \bar{D} \Sigma_c^* & \bar{D}^* \Sigma_c^* \\
g_i & 0.53 - i0.07 & 0.08 - i0.07 & 2.81 - i0.07 & 0.12 - i0.10 & 0.11 - i0.51 \\
|g_i| & \circled{0.53} & 0.11 & \circled{2.81} & 0.16 & 0.52 \\
4481.04 + i17.38 & J/\psi N & \bar{D}^* \Lambda_c & \bar{D}^* \Sigma_c & \bar{D} \Sigma_c^* & \bar{D}^* \Sigma_c^* \\
g_i & 1.05 + i0.10 & 0.18 - i0.09 & 0.12 - i0.10 & 0.22 - i0.05 & 2.84 - i0.34 \\
|g_i| & \circled{1.05} & 0.20 & 0.16 & 0.22 & \circled{2.86} \\
\end{array}
\]
\[ T(J/\psi p)(M_{J/\psi p}) = V_p h_{K^- p} G_{J/\psi p}(M_{J/\psi p}) \]
\[ \times t_{J/\psi p \to J/\psi p}(M_{J/\psi p}), \]

\[ t_{J/\psi p \to J/\psi p} = \frac{g_{J/\psi p}^2}{M_{J/\psi p}^2 - M_R^2 + iM_R \Gamma_R} \text{ for } 2M_R \]
Observation of the $\Lambda_b^0 \rightarrow J/\psi p\pi^-$ decay

Note: No $\Delta(1232)$ !!

Peak at the same energy as in $\Lambda_b \rightarrow J/\psi K^- p$

I=1/2 if u,d quarks are in I=0 and are spectators

LHCb
A hidden-charm pentaquark state in $\Lambda_b^0 \rightarrow J/\psi p\pi^-$ decay

Wang, Chen, Geng, Li, E. O., PRD (2016)
More pentaquarks?

Dynamically generated $N^*$ and $\Lambda^*$ resonances in the hidden charm sector around 4.3 GeV

Wu, Molina, E.O. and Zou, PRC 84 (2011)

<table>
<thead>
<tr>
<th>$(I, S)$</th>
<th>$z_R$</th>
<th>$\bar{D}^*\Sigma_c$</th>
<th>$\bar{D}^*\Lambda_c^+$</th>
<th>$J/\psi N$</th>
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<td>$(1/2, 0)$</td>
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<td>$4415 - 9.5i$</td>
<td>$-0.07 + 0.05i$</td>
<td>$-0.85 + 0.02i$</td>
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<td></td>
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<tr>
<th>$(0, -1)$</th>
<th>$\bar{D}_s^*\Lambda_c^+$</th>
<th>$\bar{D}^*\Xi_c$</th>
<th>$\bar{D}^*\Xi'_c$</th>
<th>$J/\psi \Lambda$</th>
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<td>$4368 - 2.8i$</td>
<td>$1.27 - 0.04i$</td>
<td>$3.16 - 0.02i$</td>
<td>$-0.10 + 0.13i$</td>
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<td>$4547 - 6.4i$</td>
<td>$0.01 + 0.004i$</td>
<td>$0.05 - 0.02i$</td>
<td>$2.61 - 0.13i$</td>
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<td></td>
<td>$0.01$</td>
<td>$0.05$</td>
<td>$2.61$</td>
<td>$0.61$</td>
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Looking for a hidden-charm pentaquark state with strangeness $S = -1$ from $\Xi_b^- \to J/\psi K^- \Lambda$
A hidden-charm $S = -1$ pentaquark from the decay of $\Lambda_b$ into $J/\psi\eta\Lambda$ states

Feijoo, Magas, Ramos, E. O., 1512.08152
The $\Lambda_b \to J/\psi K^0 \Lambda$ reaction and a hidden-charm pentaquark state with strangeness

Lu, Wang, Xie, Geng, E. O.,
PRD (2016)
Conclusions:

There is by now wide evidence that there are two $\Lambda(1405)$ states and they are of molecular nature, mostly $K\bar{N}$ and $\pi\Sigma$.

Fits to $K\bar{N}$ data are not sufficient to pin down the properties. Consideration of $\Lambda(1405)$ production reactions is necessary.

While scattering experiments have been the main source of information so far, weak decays of $\Lambda_b$ and $\Lambda_c$ promise to be an important source in the future.

The lesson of the $\Lambda(1405)$ showing a state of clear not $qqq$ nature has been the starting point to accept that many other states can also be of more complicated nature and make us diggest better the existence of pentaquark states of molecular nature or with other configurations.

Pentaquarks, or composite states, are already here to stay and grow.
Nuclei of mesons? It is coming:
Two mesons and a baryon $\rightarrow$ A. Martinez and K. Khemchandani,
$\rightarrow$ Jido, Enyo
One vector two pseudocalars $\rightarrow$ $\Phi K K\bar{K} \rightarrow X(2175)$, Martinez, Napsuciale Khemchandani
$\rightarrow$ Oller, Alvarez Ruso, Alarcon
Multirho states $\rightarrow$ L. Roca, PRD 2010
$K^*\text{multirho states} \rightarrow$ L. Roca, J. Yamagata, PRD 2010
\[ \frac{\rho_1}{\rho_2} = \frac{f_2}{f_4} = \frac{K_2^*}{K_4^*} = \frac{K_3^*}{K_5^*} \]
General recommendation: If one wants to know where the poles of the resonances are, better use reactions where the resonances are produced and are clearly seen.
DEAR motivated work: All calculations incompatible with DEAR measurement of $K^-p$ atom (the data turned out to be incorrect)

The lowest order calculations have been improved recently in

Borasoy, Niessler and Weise, PRL (2005); Oller, Prades, Verbeni, PRL (2005); Oller (2006); Borasoay, Meissner and Niessler (2006)

Common features: two poles for the $\Lambda(1405)$, one around 1420 MeV with narrow width ($\sim 30 MeV$). The second one at lower energies, wider but changes much from model to model. Observation of the $\Lambda(1405)$ with different shapes in different reactions should further constraint the models.

Some differences: predictions for the scattering lengths. More experimental work on $K^-p$ atoms is needed.

Results with lowest order Lagrangian compatible with theoretical band determined by Borasoay et al.
I=1 amplitudes in the standard approach to scattering data
We allow the regulators $a_i$ to vary a bit with respect to the original values.

We respect unitarity in coupled channels. $b(W), c(W)$ are complex numbers for each energy, $\alpha_i$ are parameters of the potential.

Why we improve over analysis of Moriya et al.?

Moriya et al.

\[
t_I(m) = C_I(W) e^{i\Delta \phi_I} B_I(m)
\]

where $C_I(W)$ is a weight factor, $\Delta \phi_I$ a phase.

$C_I(W)$ is real, $\Delta \phi_I$ is energy independent.

But amplitudes are not BW, and unitarity is lost when two BW amplitudes are summed.
This could be $3/2^-, 5/2^+$

All fits prefer to have the wide state with negative parity, interfering with the $\Lambda(1405)$.
Observation of the $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ decay

Note: No $\Delta(1232)$ !

$\Delta_b \rightarrow J/\psi K^- p$

I=1/2 if u,d quarks are in I=0 and are spectators

Triangle singularity
Is different now