Angular distribution of exclusive dielectron production in pion-nucleon collisions

Enrico Speranza

with Miklós Zétényi and Bengt Friman

Polarization and dilepton anisotropy in pion-nucleon collisions


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Introduction

$\pi N \rightarrow R \rightarrow Ne^+e^-$

- Elementary reactions are important for nuclear collisions (W. Przygoda talk)
- HADES studied pion induced reactions at $\sqrt{s} = 1.49$ GeV
  Nearby resonances: N(1440), N(1520), N(1535), Δ(1600)

The goal:

Angular distributions of the dilepton

Information on the polarization states of the resonance

Help disentangle different sources in hadronic and nuclear collisions
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Study the anisotropy coefficient

\[
\frac{d\sigma}{dM \, d\cos\theta_{\gamma^*} \, d\cos\theta_e} \propto \Sigma_\perp (1 + \cos^2\theta_e) + \Sigma_\parallel (1 - \cos^2\theta_e)
\]

\[
\propto \mathcal{N} (1 + \lambda_\theta(\theta_{\gamma^*}, M) \cos^2\theta_e)
\]

\[
\lambda_\theta(\theta_{\gamma^*}, M) = \frac{\Sigma_\perp - \Sigma_\parallel}{\Sigma_\perp + \Sigma_\parallel}
\]

\(\lambda_\theta(\theta_{\gamma^*}, M)\) contains information on the polarization of the virtual photon and hence on the quantum numbers of the baryon resonance.
Study the anisotropy coefficient

\[
\frac{d\sigma}{dM \, d \cos \theta_{\gamma^*} \, d \cos \theta_e} \propto \Sigma_{\perp} (1 + \cos^2 \theta_e) + \Sigma_{\parallel} (1 - \cos^2 \theta_e) \\
\propto \mathcal{N} (1 + \lambda_\theta(\theta_{\gamma^*}, M) \cos^2 \theta_e)
\]

\[\lambda_\theta(\theta_{\gamma^*}, M) = \frac{\Sigma_{\perp} - \Sigma_{\parallel}}{\Sigma_{\perp} + \Sigma_{\parallel}}\]

\[\lambda_\theta(\theta_{\gamma^*}, M)\] contains information on the polarization of the virtual photon and hence on the quantum numbers of the baryon resonance.
Angular momentum coupling

\[ \vec{J}_R = \vec{L} + \vec{S}_N \]
\[ M_R = M_L + M_N = M_N = \pm \frac{1}{2} \]

\[ |\pi(p); N(-p)\rangle \propto \sum_{lm} Y_{LM}^*(\theta, \phi) |LM\rangle \quad (Y_{LM}(\theta = 0, \phi) = 0 \text{ for } M_L \neq 0) \]

- Resonance with \( J_R = \frac{1}{2} \) \( \Rightarrow \) all the states are equally populated
  \( \Rightarrow \) Isotropic distribution in \( \theta, \gamma^* \)

- Resonance with \( J_R \geq \frac{3}{2} \) \( \Rightarrow \) not all the states are populated
  \( \Rightarrow \) Anisotropic distribution in \( \theta, \gamma^* \)
Examples

- Drell-Yan process $q\bar{q} \rightarrow e^+e^-$
  
  $$
  \frac{d\sigma}{d\Omega_e} \sim 1 + \cos^2 \theta_e
  $$

  $\lambda_\theta = +1$ Virtual photon is completely transverse polarized

- Pion annihilation process $\pi^+\pi^- \rightarrow e^+e^-$
  
  $$
  \frac{d\sigma}{d\Omega_e} \sim 1 - \cos^2 \theta_e
  $$

  $\lambda_\theta = -1$ Virtual photon is completely longitudinal polarized


- $X(3872)$ decay (CDF Collaboration, PRL 98 (2007) 132002)

Derive constraints on spin, parity and charge conjugation parity of the $X(3872)$ by comparing measured angular distributions of the decay products with predictions for different $J^{PC}$ hypothesis
\[ \pi N \rightarrow R \rightarrow Ne^+e^- \text{ cross section} \]

\[ d\sigma = \frac{1}{4F} \sum |\mathcal{M}|^2 d\Phi^{(3)} \]

- **Matrix element**
  \[ \sum |\mathcal{M}|^2 = H^{\mu\nu} L_{\mu\nu} \]

- **Lepton tensor**
  \[ L_{\mu\nu} = 2q^2 \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) - 8 \left( p_{e\mu} - \frac{q_\mu}{2} \right) \left( p_{e\nu} - \frac{q_\nu}{2} \right) \]

- **Invariant flux**
  \[ F = \sqrt{(p_\pi \cdot p_N) - m_\pi^2 m_N^2} \]

- **Three-body phase space**
  \[ d\Phi^{(3)} = \int dM^2 \frac{1}{2\pi} \frac{1}{16\pi^2} \frac{p_\rho}{\sqrt{s}} d\Omega_{\gamma^*} \frac{1}{16\pi^2} \frac{p_e^*(M^2, m_e^2, m_e^2)}{M} d\Omega_e \]
Models

- Gauge invariant vector meson dominance for $\rho^0 - \gamma^*$ coupling

$$\mathcal{L}_{\rho\gamma} = -\frac{e}{2g_\rho} F^{\mu\nu} \rho^0_{\mu\nu}$$

- Interactions for spin-1/2 resonances with $\pi$ and $\rho$

$$\mathcal{L}_{R_{1/2} N\pi} = -\frac{g_{RN\pi}}{m_\pi} \bar{\psi}_R \Gamma_{\gamma^\mu} \bar{\tau} \psi_N \cdot \partial_\mu \bar{\pi} + \text{h.c.}$$

$$\mathcal{L}_{R_{1/2} N\rho} = \frac{g_{RN\rho}}{2m_\rho} \bar{\psi}_R \bar{\tau} \sigma^{\mu\nu} \tilde{\Gamma} \psi_N \cdot \bar{\rho}_{\mu\nu} + \text{h.c.}$$

$\Gamma = \gamma_5$ for $J^P = 1/2^+$, $\Gamma = 1$ for $J^P = 1/2^-$
Consistent interactions for higher spin resonances

- Lower spin components of the Rarita-Schwinger fields should not contribute
- Lagrangians must be invariant under the transformations:
  \[ \psi_{\mu} \rightarrow \psi_{\mu} + i\partial_{\mu}\chi \]
  \[ \psi_{\mu\nu} \rightarrow \psi_{\mu\nu} + \frac{i}{2}(\partial_{\mu}\chi_{\nu} - \partial_{\nu}\chi_{\mu}) \]

- Gauge invariant operators:
  \[ G_{\mu,\nu} = i(\partial_{\mu}\psi_{\nu} - \partial_{\nu}\psi_{\mu}) \]
  \[ G_{\mu\nu,\lambda\rho} = -\partial_{\mu}\partial_{\nu}\psi_{\lambda\rho} - \partial_{\lambda}\partial_{\rho}\psi_{\mu\nu} + \frac{1}{2}(\partial_{\mu}\partial_{\lambda}\psi_{\nu\rho} + \partial_{\mu}\partial_{\rho}\psi_{\nu\lambda} + \partial_{\nu}\partial_{\lambda}\psi_{\mu\rho} + \partial_{\nu}\partial_{\rho}\psi_{\mu\lambda}) \]

\(\text{RN}_\pi\) vertex
\[ \mathcal{L}_{R3/2N\pi} = \frac{ig_{\text{RN}_\pi}}{m_{\pi}^2} \bar{\Psi}_{R}^{\mu}\bar{\gamma}_N^{\nu}\gamma_\rho \tilde{\psi}_{\mu\nu}\cdot \bar{\rho}_{\mu\rho} \]
\[ \mathcal{L}_{R5/2N\pi} = -\frac{g_{\text{RN}_\pi}}{m_{\pi}^4} \bar{\Psi}_{R}^{\mu\nu}\bar{\gamma}_N^{\rho}\gamma_\lambda \tilde{\psi}_{\mu\nu}\cdot \bar{\rho}_{\mu\nu} \]

\(\psi_{\mu} = \gamma^\nu G_{\mu,\nu}, \psi_{\mu\nu} = \gamma^\lambda\gamma^\rho G_{\mu\nu,\lambda\rho}. \gamma = \gamma_5 \) for \(J^P = 3/2^-\), \(5/2^+\) and \(\gamma = 1\) otherwise. \(\bar{\gamma} = \gamma_5\gamma.\)

The model presents two other kinds of \(RN\rho\) vertices

Anistropy coefficients

\[ \sqrt{s} = 1.49 \text{ GeV} \quad M = 0.5 \text{ GeV} \]

\[ |A_s|^2 \]

\[ |A_s^i + A_u^i|^2 \]

- Spin and parity of the intermediate resonance is reflected in a characteristic angular dependence of \( \lambda_\theta \)
- The \( u \)-channel is negligible on-shell
Angular distributions

\[ \sqrt{s} = 1.49 \text{ GeV} \quad M = 0.5 \text{ GeV} \]

\[ |A_s^i + A_u^i|^2 \]

- \( N(1440) \) and \( N(1520) \) are dominant
- Coupling constants are determined from decay rates (PDG)
N(1440) and N(1520)

\[ \sqrt{s} = 1.49 \text{ GeV} \quad M = 0.5 \text{ GeV} \]

\[ |A^s_i + A^u_i|^2 \]

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- Relative phase of the couplings is unknown. It can be determined by experiments
- \( \lambda_\theta \) does not depend strongly on the relative phase of the couplings
Invariant mass dependence for $\lambda_\theta$

\[ \sqrt{s} = 1.49 \text{ GeV} \]

\[ \sum_i |A_s^i + A_u^i|^2 \quad N(1440), N(1520) \]

- $\lambda_\theta = \frac{\Sigma_\perp - \Sigma_\parallel}{\Sigma_\perp + \Sigma_\parallel} \to 1$ as $M \to 0$ (real photon limit $\Sigma_\parallel \to 0$)
- Rough binning both in $M$ and $\theta_{\gamma^*}$ would be sufficient
Conclusions

Summary

▶ Anisotropy coefficient as a tool to understand which baryon resonance contributes
▶ We used consistent interactions for higher spin resonances
▶ Dependence on the channel for $\lambda_\theta$
▶ N(1440) and N(1520) are dominant at $\sqrt{s} = 1.49\,\text{GeV}$. The relative phase of the couplings does not influence strongly the anisotropy coefficient, but it does affect the angular distributions
▶ Adding N(1535) or $\Delta(1600)$ does not influence strongly $\lambda_\theta$

Outlook

▶ Add non-resonant (background) terms
▶ Study the hadronic final state ($\pi\pi N$)
▶ Study polarization effects in hot and dense nuclear systems
▶ Rough binning both in $M$ and $\theta_{\gamma^*}$ would be sufficient to extract information on polarization
BACKUP
Angular distribution and spin density matrix

\[ \frac{d\sigma}{dM \, d\cos \theta^\gamma \, d\cos \theta_e \, d\phi_e} \propto \sum_{\text{pol}} |\mathcal{M}|^2 \propto \sum_{\lambda, \lambda'} \rho^{\text{had}}_{\lambda, \lambda'} \rho^{\text{lep}}_{\lambda', \lambda} \]

Spin density matrices:
\[ \rho^{\text{had}}_{\lambda, \lambda'} = \frac{e^2}{k^4} \epsilon^\mu(k, \lambda) H_{\mu \nu} \epsilon^{\nu}(k, \lambda')^* \]
\[ \rho^{\text{lep}}_{\lambda', \lambda} = e^\mu(k, \lambda') L_{\mu \nu} \epsilon^{\nu}(k, \lambda)^* \]

\[ \rho^{\text{lep}}_{\lambda', \lambda} = k^2 \begin{pmatrix} 1 + \cos^2 \theta_e & \sqrt{2} \cos \theta_e \sin \theta_e e^{i\phi_e} & \sin^2 \theta_e e^{2i\phi_e} \\ \sqrt{2} \cos \theta_e \sin \theta_e e^{-i\phi_e} & 2(1 - \cos^2 \theta_e) & \sqrt{2} \cos \theta_e \sin \theta_e e^{i\phi_e} \\ \sin^2 \theta_e e^{-2i\phi_e} & \sqrt{2} \cos \theta_e \sin \theta_e e^{-i\phi_e} & 1 + \cos^2 \theta_e \end{pmatrix} \]

\[ \sum_{\text{pol}} |\mathcal{M}|^2 \propto (1 + \cos^2 \theta_e) (\rho^{\text{had}}_{-1, -1} + \rho^{\text{had}}_{1, 1}) + (1 - \cos^2 \theta_e) 2 \rho^{\text{had}}_{0, 0} \]

\[ + \sqrt{2} \cos \theta_e \sin \theta_e \left[ e^{i\phi_e} (\rho^{\text{had}}_{-1, 0} + \rho^{\text{had}}_{0, 1}) + e^{-i\phi_e} (\rho^{\text{had}}_{1, 0} + \rho^{\text{had}}_{0, -1}) \right] \]

\[ + \sin^2 \theta_e (e^{2i\phi_e} \rho^{\text{had}}_{-1, 1} + e^{-2i\phi_e} \rho^{\text{had}}_{1, -1}) \].

\[ \frac{d\sigma}{dM \, d\cos \theta^\gamma \, d\cos \theta_e \, d\phi_e} \propto 1 + \lambda_\theta \cos^2 \theta_e + \lambda_{\theta \phi} \sin 2\theta_e \cos \phi_e + \lambda_\phi \sin^2 \theta_e \cos 2\phi_e \]

\[ \lambda_\theta = \frac{\rho^{\text{had}}_{-1, -1} + \rho^{\text{had}}_{1, 1} - 2 \rho^{\text{had}}_{0, 0}}{\rho^{\text{had}}_{-1, -1} + \rho^{\text{had}}_{1, 1} + 2 \rho^{\text{had}}_{0, 0}}, \quad \lambda_{\theta \phi} = \sqrt{2} \text{Re}(\rho^{\text{had}}_{-1, 0} + \rho^{\text{had}}_{0, 1}) \rho^{\text{had}}_{-1, -1} + \rho^{\text{had}}_{1, 1} + 2 \rho^{\text{had}}_{0, 0}, \quad \lambda_\phi = \frac{2 \text{Re}(\rho^{\text{had}}_{-1, 1})}{\rho^{\text{had}}_{-1, -1} + \rho^{\text{had}}_{1, 1} + 2 \rho^{\text{had}}_{0, 0}} \]
$$\sqrt{s} = 1.49 \text{ GeV} \quad M = 0.5 \text{ GeV}$$

$$|A_s^i + A_u^i|^2$$

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▶ $\lambda_\theta$ does not depend strongly on the relative phase of the couplings
Adding $\Delta(1600)$

$\sqrt{s} = 1.49 \text{ GeV} \quad M = 0.5 \text{ GeV}$

$|A^i_s + A^i_u|^2$

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$\lambda_\theta$ does not depend strongly on the relative phase of the couplings